B.Sc. Lecturered By Dr. H.K. Yadav.
Deptt. of Mathematics Marwari College, DBG
B.Sc. part-I (Sub), Date - 02-05-2020.
Algebra (Group)

Q.1. Show that the identity element in a group is unique.

Proof: Let e_1 and e_2 be two distinct identities of the group G. Then, by definition of identity, we have

$e_1 * e_2 = e_1$ [$\because e_2$ is identity]

and $e_1 * e_2 = e_2$ [$\because e_1$ is identity]

Hence, it follows that $e_1 = e_2$

$\Rightarrow$ Identity is unique.

Hence, the identity element in a group is unique.

Q.2. Show that the set of all integers form an abelian group with respect to addition.

Proof: Let $a, b \in Z$. Then $a+b+1 \in Z$. So, $a*b \in Z$.

$\therefore$ Z is closed under the composition *

Next $(a*b)*c = (a+b+1)*c$
$= a+b+1+c+1 = a+b+c+2$

$a*(b*c) = a*(b+c+1)$
$= a+b+c+1+1 = a+b+c+2.$

$\therefore (a*b)*c = a*(b*c), \forall a,b,c \in Z.$

$\therefore$ Associative law holds.

Let $e \in Z$ be such that $a*e = e*a = a$

Then, $a+e+1 = a$ i.e. $e = -1 \in Z$.

$\therefore$ The identity element is -1.

Let, for $a \in Z$ there exists b such that
$a*b = b*a = e = -1$.

Then, $a+b+1 = -1$, i.e. $b = -a-2 \in Z$.

$\therefore$ The inverse element of $a \in Z$ is $-a-2$.

Also, $a*b = a+b+1 = b+a+1$
$= b*a$

$\therefore$ Commutative law holds.

$\therefore (Z, *)$ is an abelian group.

Algebra B.sc - part - I (Sub) Date - 07 - 05 - 2020

Q.3. Define a ring with an example.

Soln. Ring :- Let R be a non-empty set. An algebraic structure $(R, '+', '\cdot')$ together with two binary operations addition and multiplication for all $a, b \in R$ is called a ring if this structure satisfies following properties:

I Closed under addition:

$$a+b \in R, \ \forall a, b \in R$$

(II) Associative under addition:

$$a+(b+c) = (a+b)+c, \ \forall a, b, c \in R$$

(III) Existence of identity:

$$0+a = a = a+0, \ \forall a \in R.$$

then 0 is an additive identity of R.

(IV) Existence of inverse:

there exists an element $-a \in R$ such that

$$-a+a = 0 = a+(-a), \ \forall a \in R.$$

then $-a$ is an additive inverse of a.

(V) Commutative under addition:

$$a+b = b+a, \ \forall a, b \in R$$

(VI) Closed under multiplication:

$$a \cdot b \in R, \ \forall a, b \in R$$

(VII) Associative under multiplication:

$$a \cdot (b \cdot c) = (a \cdot b) c, \ \forall a, b, c \in R.$$

(VIII) Distributive laws: [Distribution of '$\cdot$' over '+'], $\forall a, b, c \in R$.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad \text{[Left distributive]}$$

$$(b+c) \cdot a = b \cdot a + c \cdot a \quad \text{[Right distributive]}$$

For example. The set Z of all integers with respect to addition and multiplication forms a ring.

Scanned with CamScanner